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Special Issue: Using Modelling to Motivate and Teach Differential Equations
Guest Editor: Brian Winkel

Associate Editor: Greg Oates

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Aims and Scope
Mathematics pervades every area of our modern world, bringing ever more sharply into focus the responsibilities placed on those who to teach it. Prominent among these is the challenge of developing an interdisciplinary approach, so that each professional group benefits from the expertise of others.

The ***International Journal of Mathematical Education in Science and Technology*** provides a forum in which a wide range of peer-reviewed research in mathematics education can be presented, assimilated and eventually adapted to everyday needs in schools, colleges, universities, industry and commerce. The journal is especially interested in facilitating communication between researchers and practitioners, and contributions are welcome from researchers, lecturers, teachers and users of mathematics at all levels regarding the contents of courses and pedagogical approaches across all areas of STEAM (Science, Technology, Engineering, Arts and Mathematics) education. We are always delighted to receive submissions from as diverse and international a range of contributors as possible, including teachers and early-career academics.

Increasing use of technology is being made in the teaching, learning, assessment and presentation of mathematics today, and original and interesting contributions in this rapidly developing area are especially welcome. Mathematical models arising from real situations, the use of computers, digital assistants and software, new teaching aids and techniques are also an important feature. Discussion is encouraged on methods of widening applications throughout science and technology and supporting students in problem solving. We particularly welcome contributions addressing pedagogy in Biology, Chemistry, Computer science, Business, Economics and Finance, Data science, Engineering, and Physics, in addition to Mathematics itself.

Please note that the link to education/pedagogy is critical for IJMEST, and we do not publish articles which merely discuss mathematics.

A distinctive feature of the journal is the Classroom Notes section. This section is for shorter, peer-reviewed articles which are not research articles, and are often new ideas for the mathematics teacher to use in teaching.

We would be glad to receive proposals at any time for special issues in relevant areas.

Readership
Mathematicians and mathematics educators in schools, colleges, and universities.

Peer Review Policy
All submitted manuscripts are subject to initial appraisal by the Editor. If found suitable for further consideration, papers are subject to peer review by independent, anonymous expert referees. All peer review is double blind and submissions can be made online at http://mc.manuscriptcentral.com/tmes.

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For more information about the journal and guidance on how to submit, please see www.tandfonline.com/tmes

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EDITORIAL

Check for updates

Using modelling to motivate and teach differential equations

Brian Winkel

SIMIODE, Chardon, OH, USA

1. Appreciation

First, we need to say thank you to many colleagues.

We thank the Editor-in-Chief, Colin Foster, of Loughborough University, UK, for extending us at SIMIODE the invitation to organise this Special Issue and offer our deep gratitude to the Special Issues Editor Greg Oates, University of Tasmania, Australia, for all his hard work and especially for his patience and diligence.

An important role in any publication of a journal's Special Issue is the team of Guest Editors who read, assess, make suggestions/corrections, and edit submissions at several stages in the editorial process. We offer our thanks to these colleagues:

- Greg Oates, Associate Editor for Special Issues, University of Tasmania, Hobart, Australia
- Brian Winkel, Director, SIMIODE, Chardon, OH, USA – Lead Editor
- Maila Hallare, Mathematical Sciences, United States Air Force Academy, USAFA, CO, USA
- Brody Johnson, Mathematics and Statistics, Saint Louis University, St. Louis, MO, USA
- Glenn Ledder, Mathematics, Emeritus, University of Nebraska, Lincoln, NE, USA
- Yanping Ma, Mathematics, Loyola Marymount University, Los Angeles, CA, USA
- Mark Nelson, School of Mathematics and Applied Statistics, University of Wollongong, Wollongong, NSW, Australia
- Cheryl Potocki, Mathematics, Retired, Charter School of Wilmington, Wilmington, DE, USA
- Becky Sanft, Mathematics, University of North Carolina Asheville, Asheville, NC, USA

And last, but not least (indeed, of primary importance), we offer our deep appreciation to the authors of the pieces you are about to read and enjoy. These colleagues created articles about their work and experiences in the hope that readers would be inspired by and benefit from the articles. It is our hope and plan that readers of this Special Issue would find useful and immediate material to bring into their classrooms in support of the use of modelling in the teaching of differential equations.

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Our call for papers said,

> Papers in which readers can take away usable materials and methods for their teaching with specific modelling activities, strategies for teaching, data used and referenced, and classroom narratives in which the engagement of both faculty and students in the use of models is demonstrated are most desirable. These papers should include specific modelling activities (with data used and referenced), teaching pedagogies or strategies, and classroom narratives on the engagement of both faculty and students in the use of differential equations as mathematical models.

We believe the authors for this Special Issue responded in an excellent manner to this call.

2. Special issue organisation

The articles in this issue are organised into categories, with the caveat that there are many overlaps. We trust that these sections will permit you to focus on something of immediate interest to you, but encourage you to move beyond your comfort zone to consider other directions and opportunities for your teaching efforts and your students' appetites for real-world modelling.

- Single-lesson modelling activities,
- Multiple-lesson modelling activities,
- Platform-specific modelling activities,
- Beyond and before differential equation modelling.

Let us take a walk through this rich garden of resources provided by these authors.

3. Single-lesson modelling activities

As lead editor for this Special Issue, I have taken the liberty of opening this issue with a very specific and detailed lesson on modelling the fall of a column of water with data-collecting capabilities from SIMIODE YouTube channel videos. I closed the piece with information about and features of SIMIODE, the Community of Practice we offer at https://qubeshub.org/community/groups/simiode.

Corban Harwood offers a complete approach to modelling the cooling of your cup of coffee to perfection, citing some very specific accomplishments students would achieve in doing this activity. He offers modelling with data investigations and good reasons to introduce the handling of discontinuous inputs using the Laplace Transform in a natural way.

Li Zhang gives us new perspectives on the use of the classical Lanchester combat model with an opportunity to estimate parameters using data derived from a simulation produced by the effective use of random number generators in Excel as well as using dice to simulate a realistic battle between opposing forces. Finally, the Battle of Trafalgar is simulated quite effectively using these methods.

Viktoria Savatorova and Aleksei Talonov get practical with a description of how we could model a hot water heater using differential equations derived from physical laws and reasonable assumptions. The article offers a rich set of explorations based on technical water heater data and delves into numerical methods, while addressing the discontinuities of 'on-off' operation of a water heater.

Tracy Weyand uses a model for a building during an earthquake to demonstrate the notion of resonance. Examination of the response amplitude of the displacement modelled by a driven second-order differential equation is the core of the effort. This personal account of how class is conducted and student inquiry is developed is very appropriate and helpful for immediate use in any classroom.

Minchul Kang effectively offers a study of the Susceptible-Infected-Recovered (SIR) model for the spread of disease, developing several implicit analytic solution strategies from first principles on mass kinetics. The solution efforts offer new insight on the classical SIR model from a quantitative perspective.

Michael Rother presents and discusses in good detail how to model tank drainage using a simple apparatus set up of a two-litre bottle, a ruler, and a stopwatch. The notion of discharge coefficient introduces engineering reality to the project as an entrance to a discussion on parameter estimation using real data collected on the spot. Euler's method in a spreadsheet is sufficient to get across the main themes.

Forest Mannan embellishes and enhances a SIMIODE Modelling Scenario that uses real-world data to produce a model for the cooling of a beaker of water in a room with a fixed ambient temperature. The work also guides students to model cooling in an environment with varying temperatures to produce a more realistic model. An often-forgotten approach of giving students 'bad' models and asking them to critique them is used. This approach fosters good modelling skills by including the necessary questioning at each stage of the modelling process. A step-by-step procedure is given, along with a rubric for evaluation.

Jean Marie Linhart offers her students opportunities to model populations from readily available data sets. In this article, we see an ambitious effort to model the historic total global human population. The tool used is the per-capita population growth rate, which gives a ready tool and perspective in growth models, while also rendering a linear model quite often.

Elizabeth G. Arnold and her colleagues Elizabeth A. Burroughs, Owen Burroughs, and Mary Alice Carlson use physical simulations to motivate the use of differential equations in models of disease spread. Various options to block disease spread are studied: no intervention, non-vaccine intervention, or vaccine intervention. The approach is to use small amounts of dye in liquids and pH indicators as measures of spread.

4. Multiple-lesson modelling activities

Fabiana Zama considers an introduction to modelling through a microbial interaction application. The problem is in the context of wine fermentation with parameter estimation techniques for the natural death of sensitive yeast. This model is somewhat advanced with six differential equations forming a model of various populations interacting with each other.

Yves Nievergelt takes us on a harrowing journey in which we model the unfortunate attempt to fly a large airplane whose wings are perpendicular to the ground during an air show, thus removing the usual lift forces associated with horizontal flight and leading to a tragedy. The paper is rich in detail and historical studies related to flight.

Adrianne L. Jenner and Pamela M. Burrage demonstrate how to model ion channels on cell membrane surfaces that allow the passage of ions from one side of the membrane to the other. The piece offers carefully motivated background work and biological

explanations, leading to a system of two differential equations which are simulated using stochastic methods.

James Sandefur discusses how a professor could have students, using rational and exponential functions, develop models for several different density-dependent populations based on characteristics of the species. Further, modelling of sustainable harvesting is introduced, all in very rich detail.

Yanping Ma and Gail Tang provide context for relating mathematics and cultural competence, by considering a five-part bundle of activities in which students draw on their own personal and/or cultural experiences with the concept of love. Students develop love scenarios, model them, identify limitations, and offer future model revisions.

Ala J. Alnaser and Justin Hoffmeier acknowledge that students may often struggle to create and understand models offered in a course. They suggest a scaffolded sequence of labs scheduled throughout the course which are intended to teach students to construct and analyse models using differential equations in both in-class and out-of-class activities.

Glenn Fulford discusses integrating research work from epidemiology and ecology as a fruitful area for introducing modelling to students early in their studies. Coupled differential equations models from real research studies are introduced to motivate the study of mathematics.

Kerri Spooner offers modelling contexts in three courses for teaching differential equations which anchor the mathematics and act as cognitive roots for students. Students use differential equations in areas as diverse as the study of possums that threaten New Zealand cows and the analysis of the spread of zombies in a movie. Such interesting examples are used to motivate the use of methods, including graphical analysis and numerical techniques.

5. Platform-specific modelling activities

Liang Kong demonstrates how to use the language environment of *R* to study epidemic models in differential equations. Compartmental SIR modelling is the order of the day, and building these models, as well as using numerical simulations with *R*, offers students an opportunity to explore in a rich setting.

Shophika Vaithyanathasarma, France Caron, Geneviève Bistodeau-Gagnon, and Jacques Bélairuse use *Insight Maker* and *Excel* as tools to study compartmental epidemiological models to reflect the complexity of these situations with nonpharmaceutical interventions. Working from a recent study, the paper exposes students to ongoing research and encourages them to extend the results by considering the social dimensions which might be incorporated.

Alonso G. Ogueda and Padmanabhan Seshaiyer use repositories of materials and programmes, such as *Google CoLab* and *Github*, to support student learning of modelling steps: Observe, Theorise, Formulate, Analyse, Simulate, Validate, and Predict. The presentation is interspersed with macros and source code in *Python*.

6. Beyond and before differential equation modelling

James Wolper considers qualitative and numerical solutions to ordinary differential equations between the customary sections on differentiation and integration in a first

calculus course by including a section on qualitative and numerical solutions to these differential equations, making use of the Mean Value Theorem, and thus making the transition to integration more motivated. Linearisation methods provide the main vehicle for this journey.

Glenn Ledder and Stefano Manzonib demonstrate the role of differential equations in downstream coursework in an accessible manner using an optimal control problem for resource utilisation by microorganisms. This classical formulation seeks to determine an input function (control) that will meet the condition of a differential equation while maximising an integral over the course of time where the control variable is the uptake rate of a nutrient substrate. One very good point of this article is that it is often a good idea to show applications of the mathematics under study which would arise beyond the current course.

Jennifer Czocher, Elizabeth Roan, Abigail Quansah, and Andrew Baas in their thoughtful study, 'Calculus is the Study of Change, but Differential Equations Students Need Help in Quantifying It' report results on a study in using differential equations to model change from first principles by addressing students' disconnected ways of thinking about change and thereby helping faculty in ensuring that their students get the most from modelling opportunities in their differential equations coursework.

7. Conclusion

We appreciate your consideration for reading some or all of the many great articles in this Special Issue. We know that should you elect to bring some of these activities to your classroom your students will be grateful and will grow in their understanding and application of the mathematics of differential equations. Please contact the authors and let them know you appreciated their contribution and go to the SIMIODE website at https://qubeshub.org/community/groups/simiode to benefit from our many hundreds of resources in our Community of Practice.

Disclosure statement

No potential conflict of interest was reported by the author.